Evacuation

For new year's eve, the police department of Milan expects a massive inflow of people to the Duomo Square around midnight.

It is expected that from midnight to 1 a.m. the people will leave the Square and reach the closest underground stations: Cordusio, San Babila, Missori, Montenapoleone, Cairoli, Cadorna, Lanza. It is also supposed that the people will take the shortest path to reach the underground. For safety reasons, access to the Duomo underground station will be restricted.

Consider the following, simplified, street map of Milan

In the following table, we report the expected traveling time (in fraction of hours) between each pair of underground stations (including the Duomo one). Since the street map is itself sparse, we also give the table in a sparse form.

Duomo	Cordusio: 0.2	Missori: 0.2	S4: 0.2
Cordusio	S2: 0.2	S1: 0.225	Cairoli: 0.4
Missori	San Babila: 0.2125	S5: 0.35	
S6	San Babila: 0.4	S5: 0.35	Duomo: 0.4
San Babila	S5: 0.225	Montenapoleone: 0.6	
S5	S4: 0.2		
S4	Montenapoleone: 0.35	Cordusio: 0.25	
Montenapoleone	S3: 0.25	Lanza: 0.25	
S3	Cairoli: 0.05	Cordusio: 0.15	
S2	S1 : 0.0875		
Lanza	Cairoli: 0.2		
Cairoli	Cadorna : 0.2		
S1	Cadorna: 0.15		

Give a linear programming formulation for the problem of finding the shortest paths that the people will take from Duomo to reach any of the selected stations, so as to be able to suggest to the police department which routes will be, most likely, taken.

AMPL model sketch (file `evacuation.mod`)

```
# SETS

set V;
set T within V;
set E within {V,V};
set A within {V,V} :=
   E union setof{(i,j) in E} (j,i);

# PARAMS

param s symbolic in T;
param c{(i,j) in A} >= 0, default if (j,i) in E then c[j,i];
param b{h in V} =
   (if h = s then card(T)-1
    else if h in T then -1
    else 0);
```

For each edge $\{i, j\} \in E$, the set A contains the arc (i, j) and the arc (j, i).

Data (file `evacuation.dat`), optimal value: 2.9375

```
data;

set V := Duomo Cordusio SanBabila Missori Montenapoleone Cairoli Cadorna Lanza
         S1 S2 S3 S4 S5 S6;

set T := Duomo Cordusio SanBabila Missori Montenapoleone Cairoli Cadorna Lanza;

param s := Duomo;

param: E: c :=
        Duomo Cordusio          0.2,
        Duomo Missori           0.2,
        Duomo S4                0.2,
        Cordusio S2             0.2,
        Cordusio S1             0.225,
        Cordusio Cairoli        0.4,
        Missori SanBabila       0.2125,
        Missori S5              0.35,
        S6 SanBabila            0.4,
        S6 S5                   0.35,
        S6 Duomo                0.4,
        SanBabila S5            0.225,
        SanBabila Montenapoleone 0.6,
        S5 S4                   0.2,
        S4 Montenapoleone       0.35,
        S4 Cordusio             0.25,
        Montenapoleone S3       0.25,
        Montenapoleone Lanza    0.25,
        S3 Cairoli              0.05,
        S3 Cordusio             0.15,
        S2 S1                   0.0875,
        Lanza Cairoli           0.2,
        Cairoli Cadorna         0.2,
        S1 Cadorna              0.15
;
```