# Lot sizing

A company has to plan the production of 3 products, $A_1$, $A_2$, $A_3$, for a time horizon of four months, from January to April. The work days are: 22 for January, 20 for February, 23 for March, 22 for April.

Sales forecasts indicate the following maximum demand, per product and month.

| Demand | January | February | March | April |
|---|---|---|---|---|
| $A_1$ | 5300 | 1200 | 7400 | 5300 |
| $A_2$ | 4500 | 5400 | 6500 | 7200 |
| $A_3$ | 4400 | 6700 | 12500 | 13200 |

The following table reports the price of each product (Euro) and its unit production cost (Euro). It also reports the maximum number of pieces that can be produced in a single day (pieces/day), if the whole production capability of the factory is used to produce units of that product.

| Product | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| Price | 124 | 109 | 115 |
| Production cost | 75 | 53 | 65 |
| Production amount | 500 | 450 | 550 |

Inventory can be used to store units of unsold product. The inventory cost per month and unit is 3 for product $A_1$, 4 for product $A_2$, and 2 for product $A_3$. Each month, no more than 800 total units of the three products can be stored.

1. Give an integer linear programming formulation for the problem of determining a production plan that maximizes the total revenue.
2. Integrality restrictions are mandatory for this problem, since we are dealing with discrete products. In spite of this, when dealing with problems involving large quantities of product, it is often possible, when dropping the integrality constraints, to obtain solutions that are almost integer. Assess, computationally, the difference between integer and continuous optimal solutions for the original formulation.

**Optional questions**

1. Give a formulation for the variant where a minimum lot size is required whenever a product, per month, is produced, and where a fixed cost is charged, per month and product, whenever the production line for the corresponding product is active. Use the data:

| Product | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| Fixed cost | 150000 | 150000 | 100000 |
| Minimum lot size | 20 | 20 | 16 |

   Note that the decision of activating the production line of a specific product is to be taken each month.
2. Assess the effect of integrality for the variant of the problem. Do you expect the difference between the integer and continuous solutions to be larger in this case?

AMPL model sketch (file `lotsizing.mod`)

```
# SETS

set I;
param n;
set J := 1..n;

# PARAMS

param b{J};
param d{I,J};
param r{I};
param c{I};
param q{I};
param f{I};
param l{I};
param m{I};
param K;
```

Data (file `lotsizing.dat`), optimal value: [1] 2339316 - [2] 1585836

```
data;

set I := A1 A2 A3;

param n := 4;

param b :=
1       22
2       20
3       23
4       22
;

param d:        1       2       3       4 :=
        A1      5300    1200    7400    5300
        A2      4500    5400    6500    7200
        A3      4400    6700    12500   13200
;

param:  r       c       q       f       l       m :=
A1      124     75      500     150000  20      3
A2      109     53      450     150000  20      4
A3      115     65      550     100000  16      2
;

param K := 800;
```